

Supplementary Material to "Patch Group Based Nonlocal Self-Similarity Prior Learning for Image Denoising"

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In this supplementary material, we provide:

1. The closed-form solution of the proposed weighted sparse coding model in the main paper.
2. Comparison of the *Patch Prior based Denoising* (PPD) method and the proposed *Patch Group Prior based Denoising* (PGPD) method.
3. More denoising results on the 20 widely used natural images.
4. The denoising results of the competing methods on the Berkeley Segmentation Data Set [1].

1. Closed-Form Solution of the Weighted Sparse Coding Problem

The weighted sparse coding problem in the main paper is:

$$\min_{\alpha} \|\mathbf{y} - \mathbf{D}\alpha\|_2^2 + \|\mathbf{w}^T \alpha\|_1. \quad (1)$$

Since \mathbf{D} is an orthonormal matrix, problem (1) is equivalent to

$$\min_{\alpha} \|\mathbf{D}^T \mathbf{y} - \alpha\|_2^2 + \|\mathbf{w}^T \alpha\|_1. \quad (2)$$

For simplicity, we denote $\mathbf{z} = \mathbf{D}^T \mathbf{y}$. Since $\mathbf{w}_i = c * 2\sqrt{2}\sigma^2 / (\lambda_i + \varepsilon)$ is positive (please refer to Eq. (16) in the main paper), problem (2) can be written as

$$\min_{\alpha} \sum_{i=1}^{p^2} ((\mathbf{z}_i - \alpha_i)^2 + \mathbf{w}_i |\alpha_i|). \quad (3)$$

The problem (3) is separable w.r.t. α_i and can be simplified to p^2 scalar minimization problems

$$\min_{\alpha_i} (\mathbf{z}_i - \alpha_i)^2 + \mathbf{w}_i |\alpha_i|, \quad (4)$$

where $i = 1, \dots, p^2$. Taking derivative of α_i in problem (4) and setting the derivative to be zero. There are two cases for the solution.

(a) If $\alpha_i \geq 0$, we have

$$2(\alpha_i - \mathbf{z}_i) + \mathbf{w}_i = 0. \quad (5)$$

The solution is

$$\hat{\alpha}_i = \mathbf{z}_i - \frac{\mathbf{w}_i}{2} \geq 0. \quad (6)$$

So $\mathbf{z}_i \geq \frac{\mathbf{w}_i}{2} > 0$, and the solution $\hat{\alpha}_i$ can be written as

$$\hat{\alpha}_i = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}), \quad (7)$$

where $\text{sgn}(\bullet)$ is the sign function.

(b) If $\alpha_i < 0$, we have

$$2(\alpha_i - \mathbf{z}_i) - \mathbf{w}_i = 0. \quad (8)$$

The solution is

$$\hat{\alpha}_i = \mathbf{z}_i + \frac{\mathbf{w}_i}{2} < 0. \quad (9)$$

So $\mathbf{z}_i < -\frac{\mathbf{w}_i}{2} < 0$, and the solution $\hat{\alpha}_i$ can be written as

$$\hat{\alpha}_i = \text{sgn}(\mathbf{z}_i) * (-\mathbf{z}_i - \frac{\mathbf{w}_i}{2}) = \text{sgn}(\mathbf{z}_i) * (|\mathbf{z}_i| - \frac{\mathbf{w}_i}{2}). \quad (10)$$

In summary, we have the final solution of the weighted sparse coding problem (1) as

$$\hat{\alpha} = \text{sgn}(\mathbf{D}^T \mathbf{y}) \odot \max(|\mathbf{D}^T \mathbf{y}| - \mathbf{w}/2, 0), \quad (11)$$

where \odot means element-wise multiplication and $|\mathbf{D}^T \mathbf{y}|$ is the absolute value of each entry of the vector $\mathbf{D}^T \mathbf{y}$.

2. Comparison between PPD and PGPD

In this section, we compare the PSNR and visual quality of denoised images by the *Patch Prior based Denoising* (PPD) method and the *Patch Group Prior based Denoising* (PGPD) method. As can be seen from Table 1 and Figures 1-4, PGPD is much better than PPD both quantitatively and qualitatively. This validates the effectiveness of our learned PG based NSS prior. In the following sections, we will omit the results of the PPD method.

Table 1. PSNR(dB) results of PPD and PGPD on the 20 natural images.

	$\sigma = 10$		$\sigma = 20$		$\sigma = 30$		$\sigma = 40$		$\sigma = 50$		$\sigma = 75$		$\sigma = 100$	
Images	PPD	PGPD	PPD	PGPD	PPD	PGPD	PPD	PGPD	PPD	PGPD	PPD	PGPD	PPD	PGPD
Airfield	31.02	31.18	27.97	28.19	26.33	26.46	25.20	25.30	24.33	24.44	22.69	22.90	21.54	21.82
Airplane	35.87	36.00	32.59	32.69	30.62	30.80	29.21	29.44	28.10	28.38	25.90	26.39	24.35	25.01
Baboon	30.46	30.55	26.54	26.67	24.54	24.63	23.23	23.39	22.30	22.47	20.71	21.09	19.99	20.38
Barbara	33.96	34.74	30.24	31.40	27.97	29.38	26.29	27.97	24.94	26.81	22.84	24.84	22.04	23.48
Boat	33.58	33.77	30.58	30.82	28.80	29.05	27.51	27.82	26.52	26.85	24.72	25.19	23.56	24.06
C. Man	33.91	34.14	30.12	30.35	28.25	28.53	27.05	27.33	26.13	26.46	24.36	24.64	22.80	23.23
Carhouse	34.35	34.47	30.61	30.73	28.62	28.80	27.29	27.51	26.27	26.53	24.44	24.85	23.21	23.67
Couple	33.78	34.03	30.47	30.71	28.54	28.84	27.16	27.53	26.07	26.50	24.22	24.70	23.12	23.55
Elaine	32.73	32.98	31.21	31.32	30.24	30.37	29.42	29.62	28.69	28.90	27.26	27.47	26.17	26.27
Hat	35.44	35.44	31.42	31.44	29.05	29.31	27.43	27.90	26.28	26.76	24.19	24.79	22.86	23.45
Hill	33.38	33.58	30.41	30.66	28.85	29.09	27.76	28.06	26.91	27.22	25.34	25.73	24.36	24.66
House	35.61	36.56	33.18	33.85	31.62	32.24	30.32	31.02	29.17	29.93	26.81	27.81	25.13	26.17
Lake	32.86	32.98	30.00	30.09	28.30	28.38	27.03	27.15	26.05	26.20	24.19	24.49	22.94	23.36
Leaves	33.87	34.45	29.84	30.46	27.51	27.99	25.88	26.29	24.56	25.03	21.94	22.61	19.77	20.95
Lena	35.59	35.81	32.75	32.94	30.98	31.27	29.67	30.10	28.61	29.11	26.68	27.40	25.41	26.09
Man	33.91	33.98	30.52	30.60	28.72	28.86	27.53	27.73	26.63	26.86	25.01	25.36	24.00	24.33
Monarch	34.27	34.53	30.40	30.68	28.27	28.49	26.81	27.02	25.66	26.00	23.51	24.00	21.89	22.56
Paint	34.16	34.31	30.52	30.62	28.39	28.42	26.88	26.94	25.70	25.82	23.50	23.89	22.05	22.65
Peppers	34.71	34.82	32.61	32.66	31.13	31.25	29.95	30.18	28.99	29.22	27.04	27.42	25.45	25.94
Zelda	35.50	35.51	32.21	32.21	30.35	30.43	29.07	29.23	28.06	28.24	26.37	26.56	25.28	25.41
Average	33.95	34.19	30.71	30.95	28.85	29.13	27.53	27.88	26.50	26.89	24.59	25.11	23.30	23.85

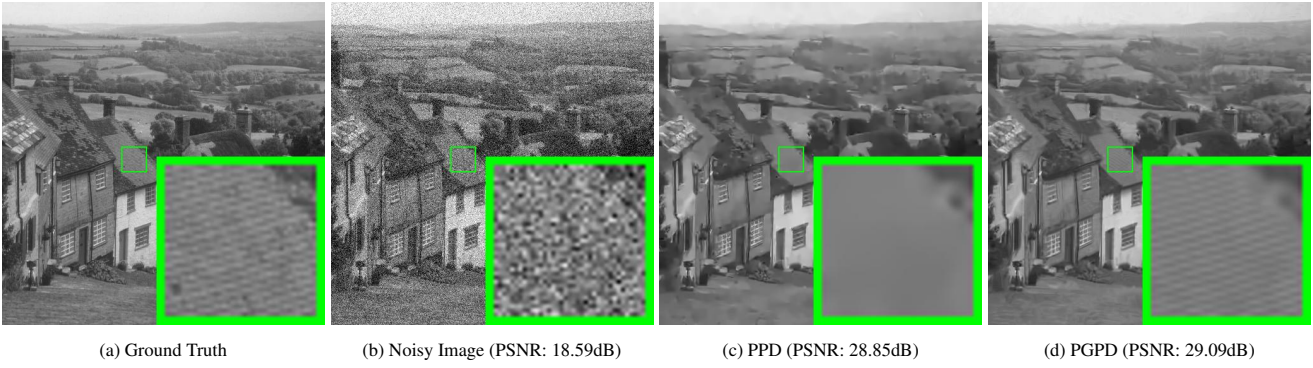


Figure 1. Denoised images of *Hill* by the PPD method and PGPD method (the standard deviation of noise is $\sigma = 30$).

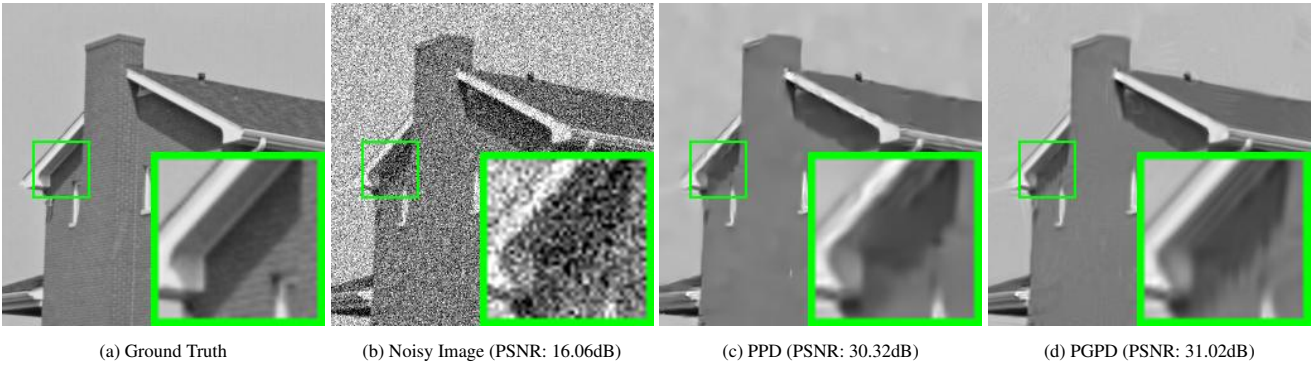


Figure 2. Denoised images of *House* by the PPD method and PGPD method (the standard deviation of noise is $\sigma = 40$).

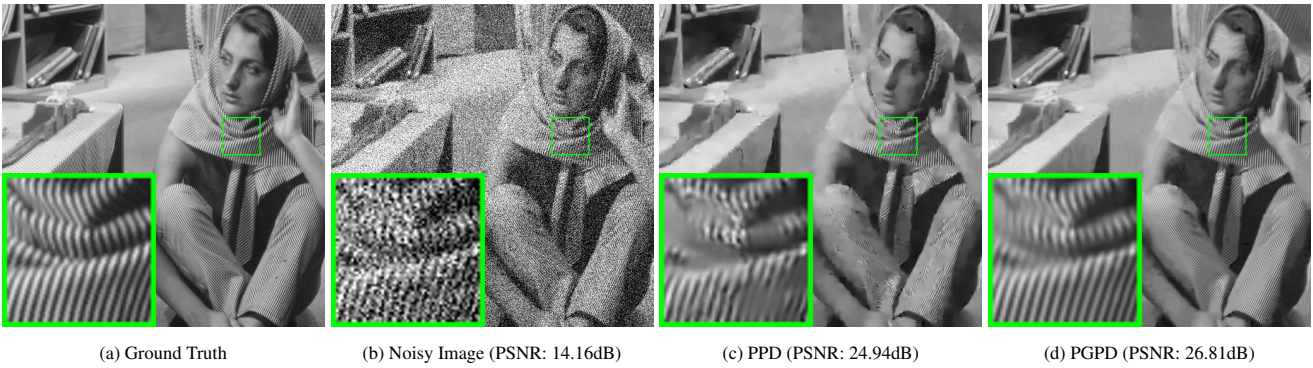


Figure 3. Denoised images of *Barbara* by the PPD method and PGPD method (the standard deviation of noise is $\sigma = 50$).

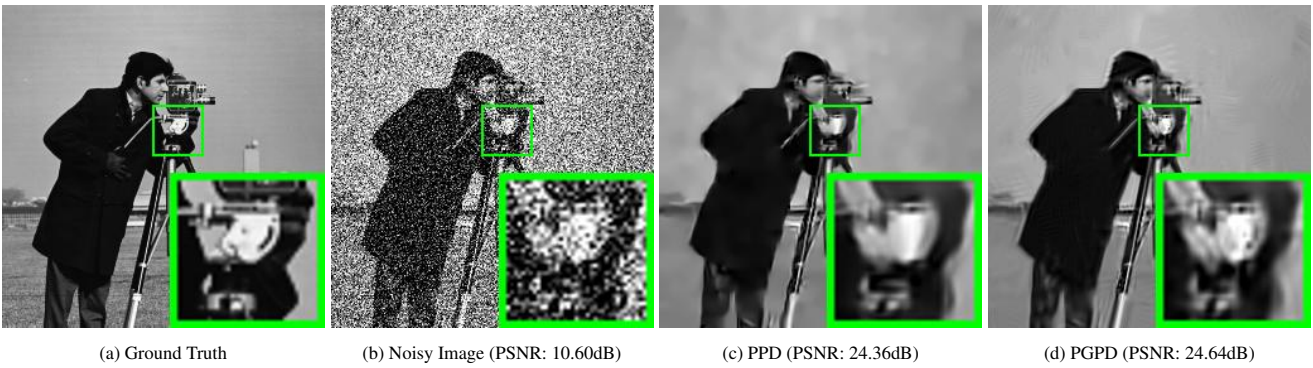


Figure 4. Denoised images of *Cameraman* by the PPD method and PGPD method (the standard deviation of noise is $\sigma = 75$).

3. More Results on the 20 Natural Images

In the main paper, we have given the PSNR results of the competing methods on the 20 widely used images when the noise standard deviations are $\sigma = 30, 40, 50, 75$. Here we give the PSNR results on these images when the noise standard deviations are $\sigma = 10, 20, 100$ in Table 2. In Figures 5-14, we give more visual comparisons of the competing methods.

Table 2. PSNR(dB) results of different denoising algorithms on the 20 natural images.

Images	$\sigma = 10$						$\sigma = 20$						$\sigma = 100$					
	BM3D	LSSC	EPLL	NCSR	WNNM	PGPD	BM3D	LSSC	EPLL	NCSR	WNNM	PGPD	BM3D	LSSC	EPLL	NCSR	WNNM	PGPD
Airfield	31.32	31.51	31.37	31.40	31.47	31.18	28.13	28.48	28.18	28.07	28.40	28.19	21.68	21.75	21.70	21.48	21.87	21.82
Airplane	35.97	36.05	35.92	36.04	36.26	36.00	32.63	32.57	32.64	32.69	32.91	32.69	24.97	24.66	24.69	24.70	25.30	25.01
Baboon	30.58	30.67	30.62	30.61	30.79	30.55	26.61	26.75	26.71	26.64	26.84	26.67	20.40	20.37	20.10	20.23	20.45	20.38
Barbara	34.98	34.99	33.65	35.01	35.51	34.74	31.78	31.60	29.85	31.78	32.19	31.40	23.62	23.55	22.21	23.20	24.37	23.48
Boat	33.92	34.03	33.67	33.92	34.09	33.77	30.88	30.92	30.71	30.79	31.01	30.82	23.97	23.87	23.86	23.68	24.10	24.06
C. Man	34.18	34.24	34.05	34.18	34.44	34.14	30.48	30.59	30.38	30.47	30.75	30.35	23.07	23.12	22.95	22.94	23.36	23.23
Carhouse	34.45	34.52	34.32	34.51	34.70	34.47	30.74	30.77	30.66	30.79	30.90	30.73	23.67	23.66	23.48	23.28	23.90	23.67
Couple	34.04	34.01	33.88	34.00	34.14	34.03	30.76	30.74	30.60	30.60	30.82	30.71	23.51	23.27	23.40	23.15	23.55	23.55
Elaine	33.36	34.36	32.90	33.83	33.76	32.98	31.48	31.74	31.25	31.47	31.44	31.32	26.21	25.96	26.33	25.85	26.29	26.27
Hat	35.40	35.56	35.23	35.45	35.75	35.44	31.55	31.48	31.41	31.48	31.66	31.44	23.34	22.90	23.33	23.08	23.35	23.45
Hill	33.62	33.68	33.49	33.69	33.79	33.58	30.72	30.73	30.50	30.65	30.81	30.66	24.58	24.48	24.62	24.36	24.75	24.66
House	36.71	36.95	35.75	36.79	36.95	36.56	33.77	34.11	33.12	33.87	34.01	33.85	25.87	25.71	25.62	25.56	26.68	26.17
Lake	33.03	33.23	33.00	33.10	33.21	32.98	30.05	30.15	30.09	30.05	30.29	30.09	23.35	23.07	23.31	23.07	23.44	23.36
Leaves	34.04	34.52	33.39	34.52	35.20	34.45	30.09	30.46	29.55	30.45	31.10	30.46	20.90	20.54	20.26	20.86	21.57	20.95
Lena	35.93	35.85	35.62	35.85	36.06	35.81	33.05	32.89	32.74	32.95	33.12	32.94	25.95	25.96	25.67	25.71	26.20	26.09
Man	33.98	34.10	34.01	34.05	34.23	33.98	30.59	30.72	30.68	30.59	30.77	30.60	24.22	23.98	24.23	24.02	24.36	24.33
Monarch	34.12	34.44	34.36	34.51	35.03	34.53	30.35	30.59	30.60	30.62	31.11	30.68	22.52	22.24	22.35	22.12	22.95	22.56
Paint	34.00	34.35	34.09	34.15	34.50	34.31	30.36	30.59	30.50	30.33	30.77	30.62	22.51	22.14	22.53	22.11	22.74	22.65
Peppers	35.01	35.13	34.83	35.03	35.06	34.82	32.76	32.61	32.61	32.66	32.81	32.66	25.85	25.63	25.75	25.40	26.20	25.94
Zelda	35.62	35.54	35.57	35.49	35.69	35.51	32.29	32.06	32.29	32.10	32.30	32.21	25.43	24.96	25.48	25.00	25.31	25.41
Average	34.21	34.39	33.99	34.31	34.53	34.19	30.95	31.03	30.75	30.95	31.20	30.95	23.78	23.59	23.59	23.49	24.04	23.85

4. Results on the Berkeley Segmentation Data Set

In this section, we give the denoising results (PSNR and visual quality) on the Berkeley Segmentation Data Set [1]. This dataset contains 500 images of size 321×481 . Since EPLL is trained on the 300 images in the training set and validation set, we use the 200 images in the testing set for comparison.

From Table 3, we can see that EPLL is comparable to BM3D and LSSC on the 200 test images. Though the proposed PGPD is trained on the Kodak PhotoCD dataset, in which the images have different resolution and statistics from the images in the Berkeley Segmentation Data Set, it still shows superiority over EPLL and other methods. In Figures 15-24, we compare the visual quality of the denoised images by the competing methods. For better visualization, the presented images are cropped to size of 321×321 .

Table 3. Average PSNR(dB) results of different denoising algorithms on the 200 test images of the Berkeley Segmentation Data Set.

Noise	BM3D	LSSC	EPLL	NCSR	WNNM	PGPD
$\sigma = 10$	33.62	33.75	33.64	33.68	33.88	33.60
$\sigma = 20$	29.86	30.02	29.96	29.89	30.11	29.89
$\sigma = 30$	27.93	28.05	28.00	27.92	28.17	27.96
$\sigma = 40$	26.58	26.75	26.71	26.58	26.88	26.73
$\sigma = 50$	25.71	25.80	25.77	25.65	25.96	25.82
$\sigma = 75$	24.22	24.18	24.18	24.04	24.42	24.30
$\sigma = 100$	23.21	23.12	23.15	23.00	23.37	23.29



(a) Ground Truth

(b) Noisy Image (PSNR: 18.59dB)

(c) BM3D (PSNR: 29.12dB)

(d) LSSC (PSNR: 29.06dB)



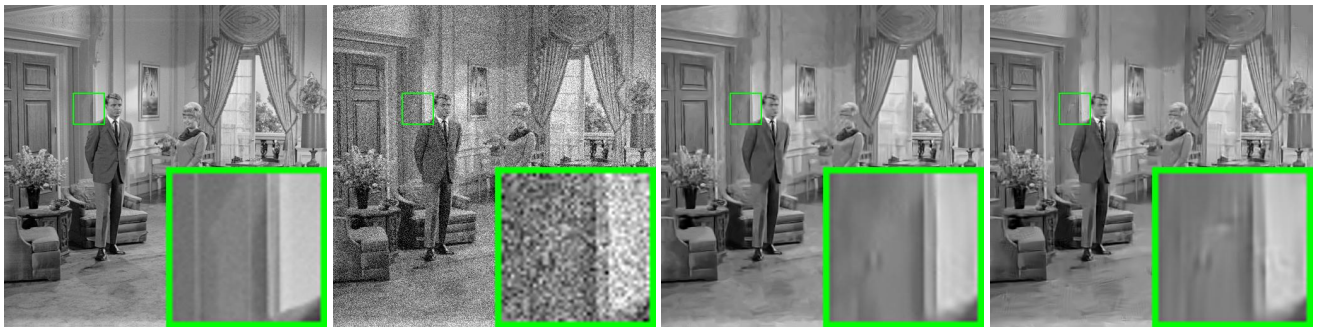
(e) EPLL (PSNR: 28.97dB)

(f) NCSR (PSNR: 28.94dB)

(g) WNNM (PSNR: 29.24dB)

(h) PGPD (PSNR: 29.05dB)

Figure 5. Denoised images of *Boat* by different methods (the standard deviation of noise is $\sigma = 30$).

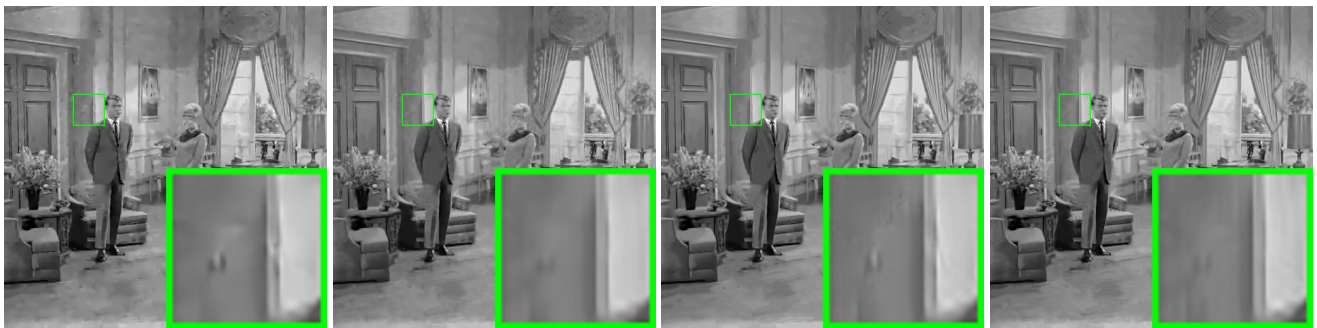


(a) Ground Truth

(b) Noisy Image (PSNR: 18.59dB)

(c) BM3D (PSNR: 28.87dB)

(d) LSSC (PSNR: 28.76dB)



(e) EPLL (PSNR: 28.69dB)

(f) NCSR (PSNR: 28.57dB)

(g) WNNM (PSNR: 28.98dB)

(h) PGPD (PSNR: 28.84dB)

Figure 6. Denoised images of *Couple* by different methods (the standard deviation of noise is $\sigma = 30$).

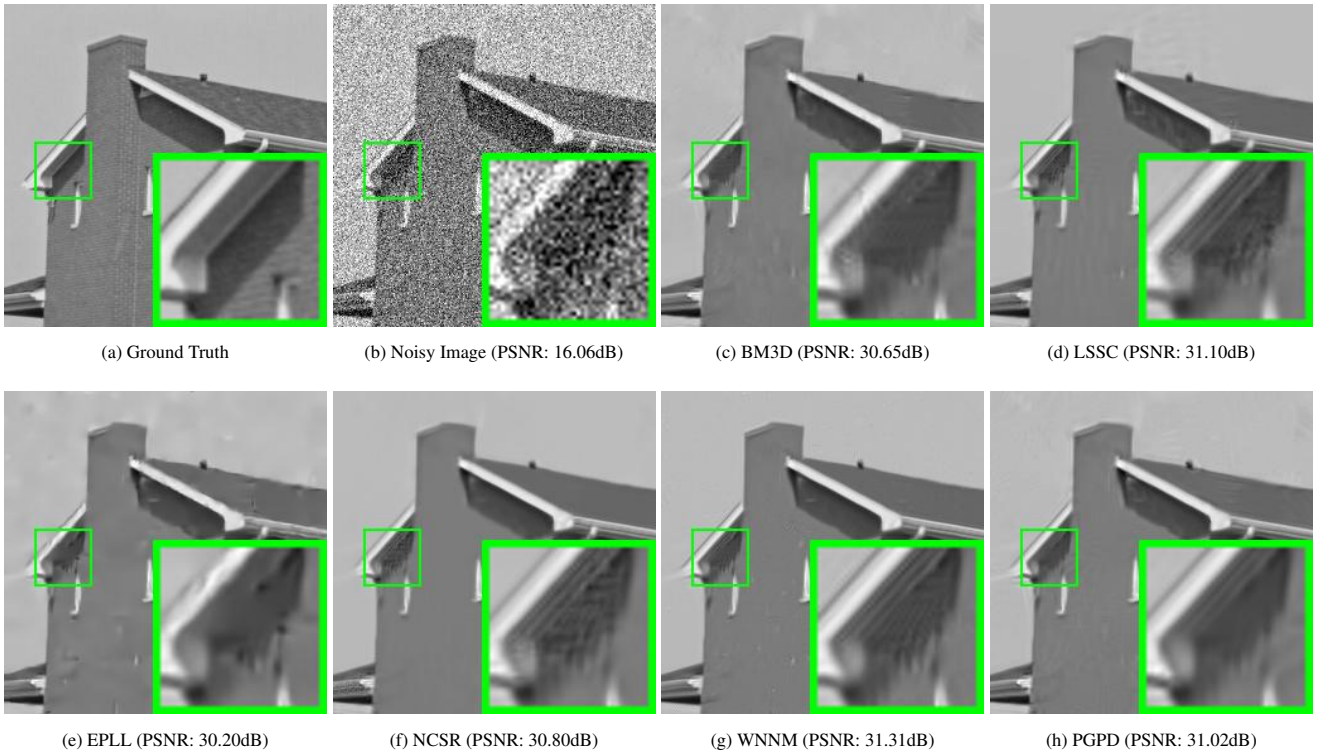


Figure 7. Denoised images of *House* by different methods (the standard deviation of noise is $\sigma = 40$).

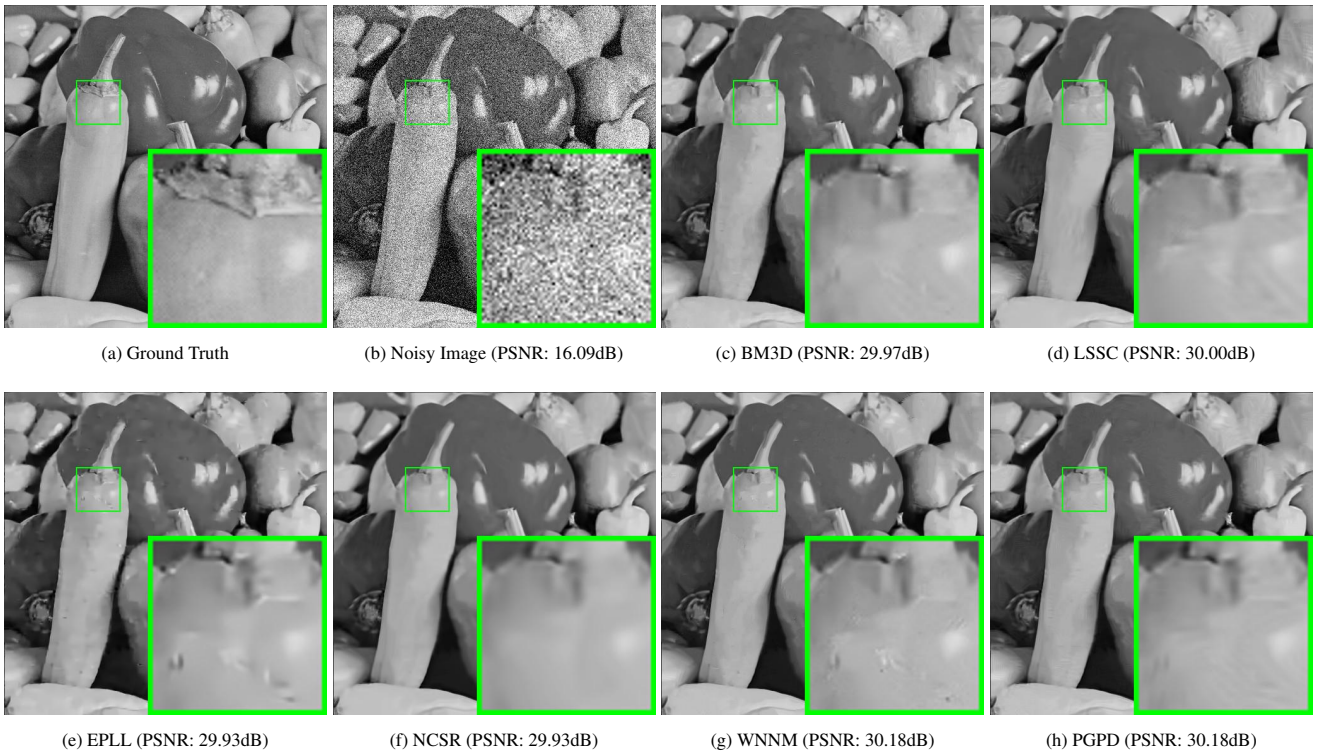


Figure 8. Denoised images of *Peppers* by different methods (the standard deviation of noise is $\sigma = 40$).



(a) Ground Truth

(b) Noisy Image (PSNR: 14.16dB)

(c) BM3D (PSNR: 24.20dB)

(d) LSSC (PSNR: 24.58dB)



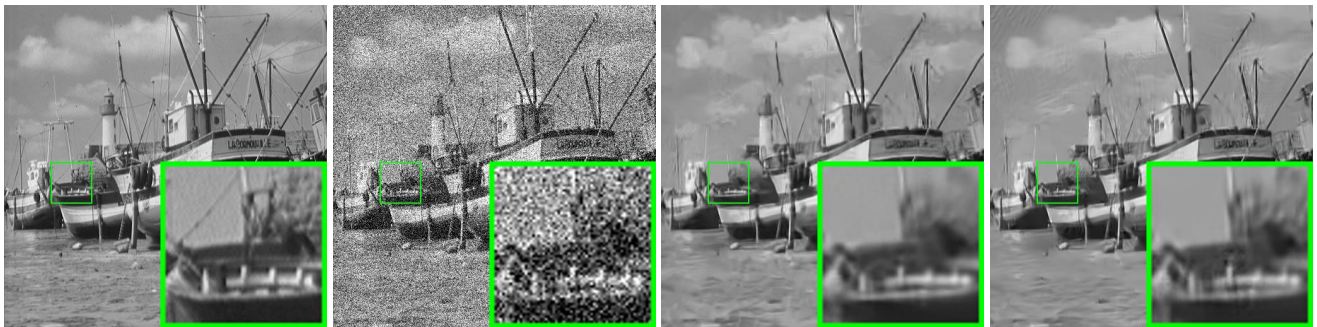
(e) EPLL (PSNR: 24.46dB)

(f) NCSR (PSNR: 24.18dB)

(g) WNNM (PSNR: 24.51dB)

(h) PGPD (PSNR: 24.44dB)

Figure 9. Denoised images of *Airfield* by different methods (the standard deviation of noise is $\sigma = 50$).



(a) Ground Truth

(b) Noisy Image (PSNR: 14.16dB)

(c) BM3D (PSNR: 26.78dB)

(d) LSSC (PSNR: 26.77dB)



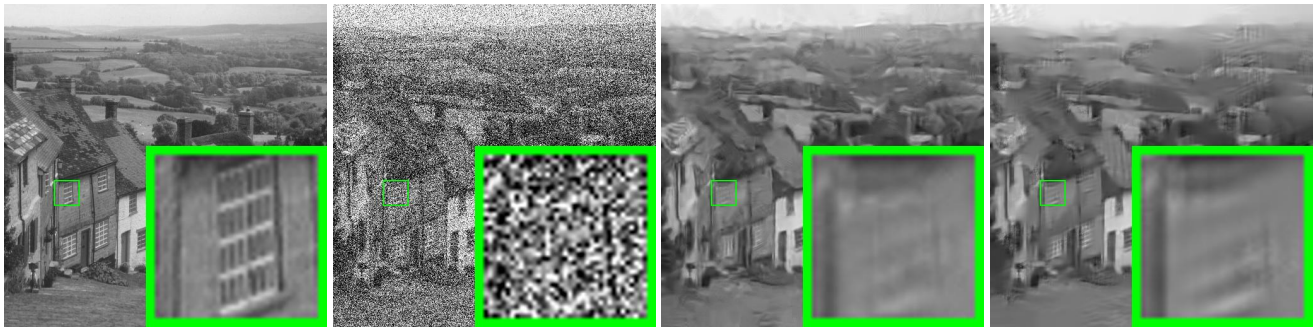
(e) EPLL (PSNR: 26.74dB)

(f) NCSR (PSNR: 26.67dB)

(g) WNNM (PSNR: 26.97dB)

(h) PGPD (PSNR: 26.85dB)

Figure 10. Denoised images of *Boat* by different methods (the standard deviation of noise is $\sigma = 50$).

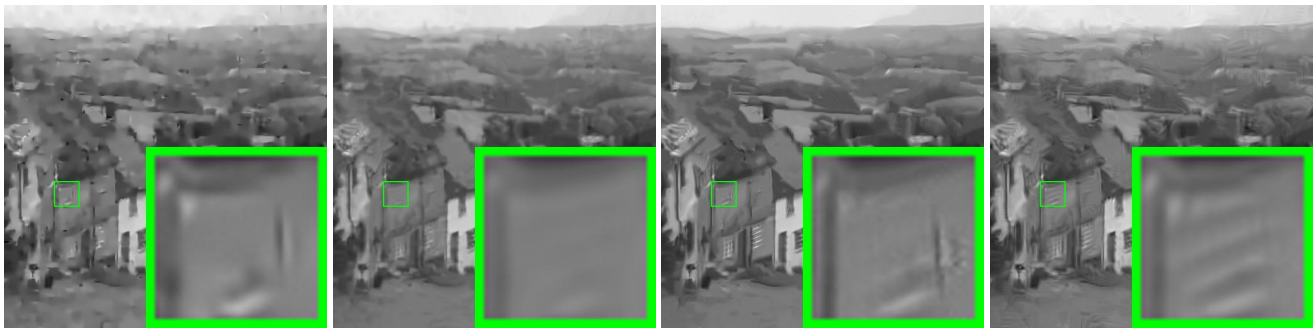


(a) Ground Truth

(b) Noisy Image (PSNR: 10.63dB)

(c) BM3D (PSNR: 25.68dB)

(d) LSSC (PSNR: 25.57dB)



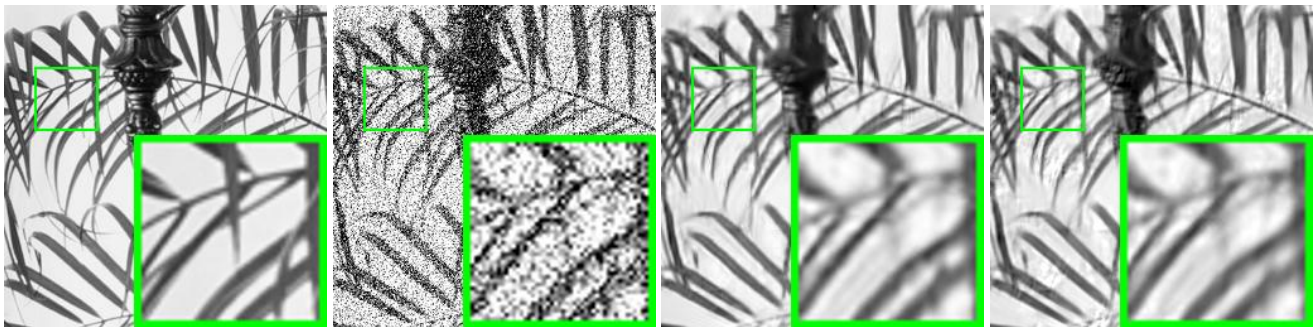
(e) EPLL (PSNR: 25.60dB)

(f) NCSR (PSNR: 25.40dB)

(g) WNNM (PSNR: 25.88dB)

(h) PGPD (PSNR: 25.73dB)

Figure 11. Denoised images of *Hill* by different methods (the standard deviation of noise is $\sigma = 75$).

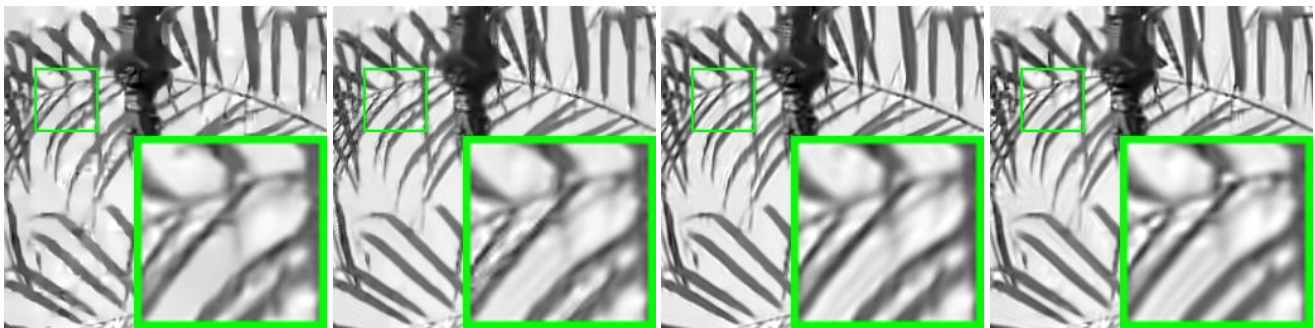


(a) Ground Truth

(b) Noisy Image (PSNR: 10.63dB)

(c) BM3D (PSNR: 22.49dB)

(d) LSSC (PSNR: 22.17dB)



(e) EPLL (PSNR: 22.12dB)

(f) NCSR (PSNR: 22.60dB)

(g) WNNM (PSNR: 23.06dB)

(h) PGPD (PSNR: 22.61dB)

Figure 12. Denoised images of *leaves* by different methods (the standard deviation of noise is $\sigma = 75$).

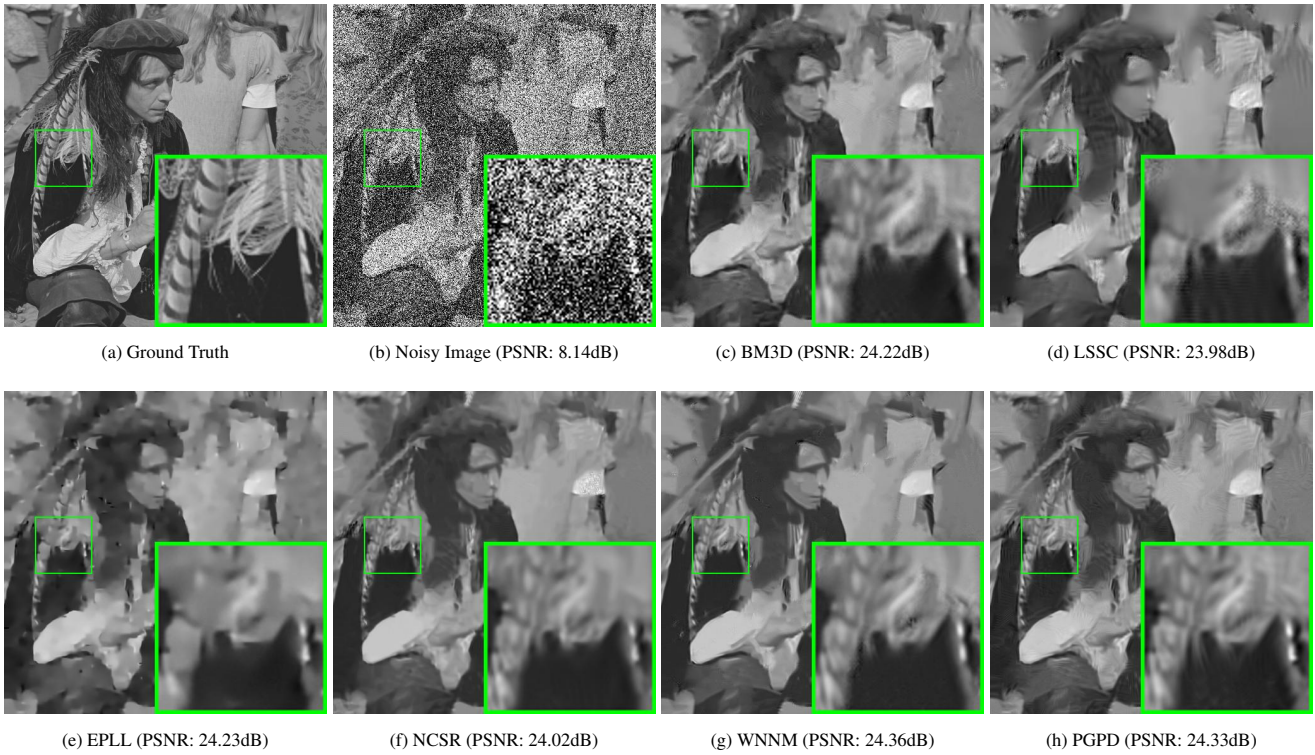


Figure 13. Denoised images of *Man* by different methods (the standard deviation of noise is $\sigma = 100$).

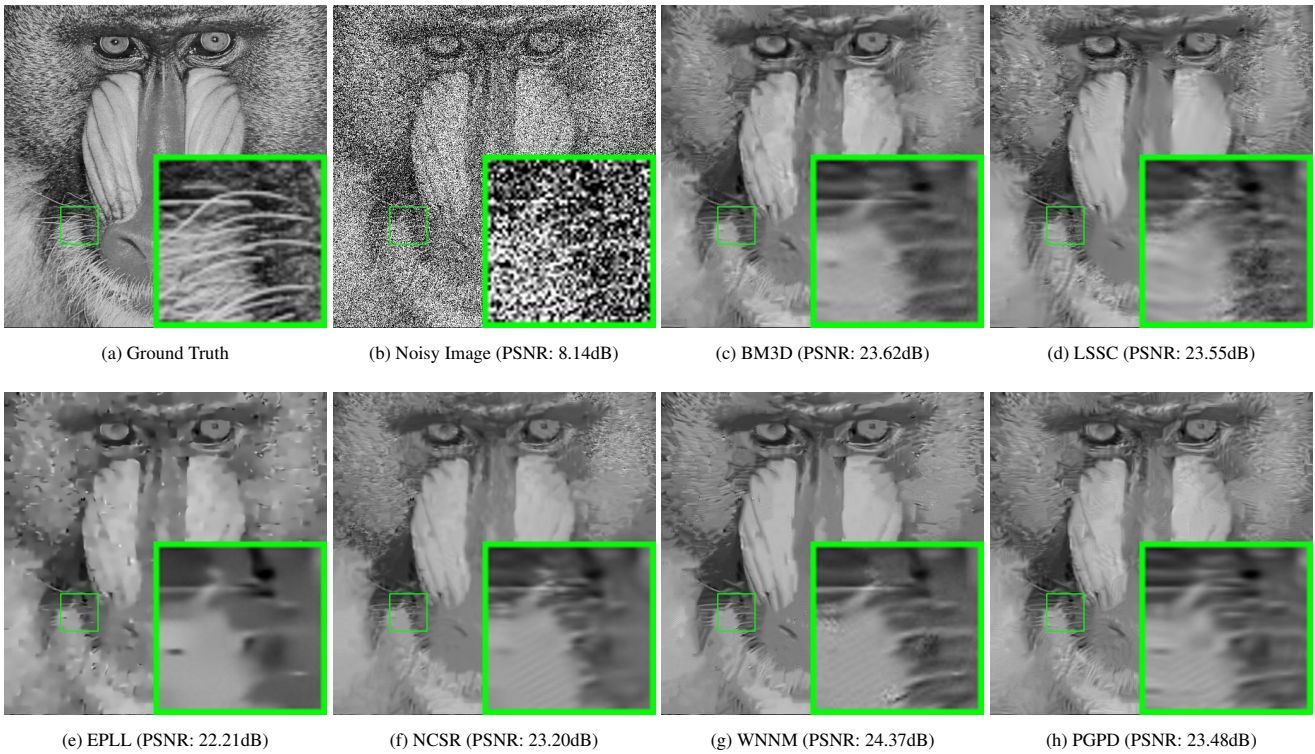


Figure 14. Denoised images of *Baboon* by different methods (the standard deviation of noise is $\sigma = 100$).

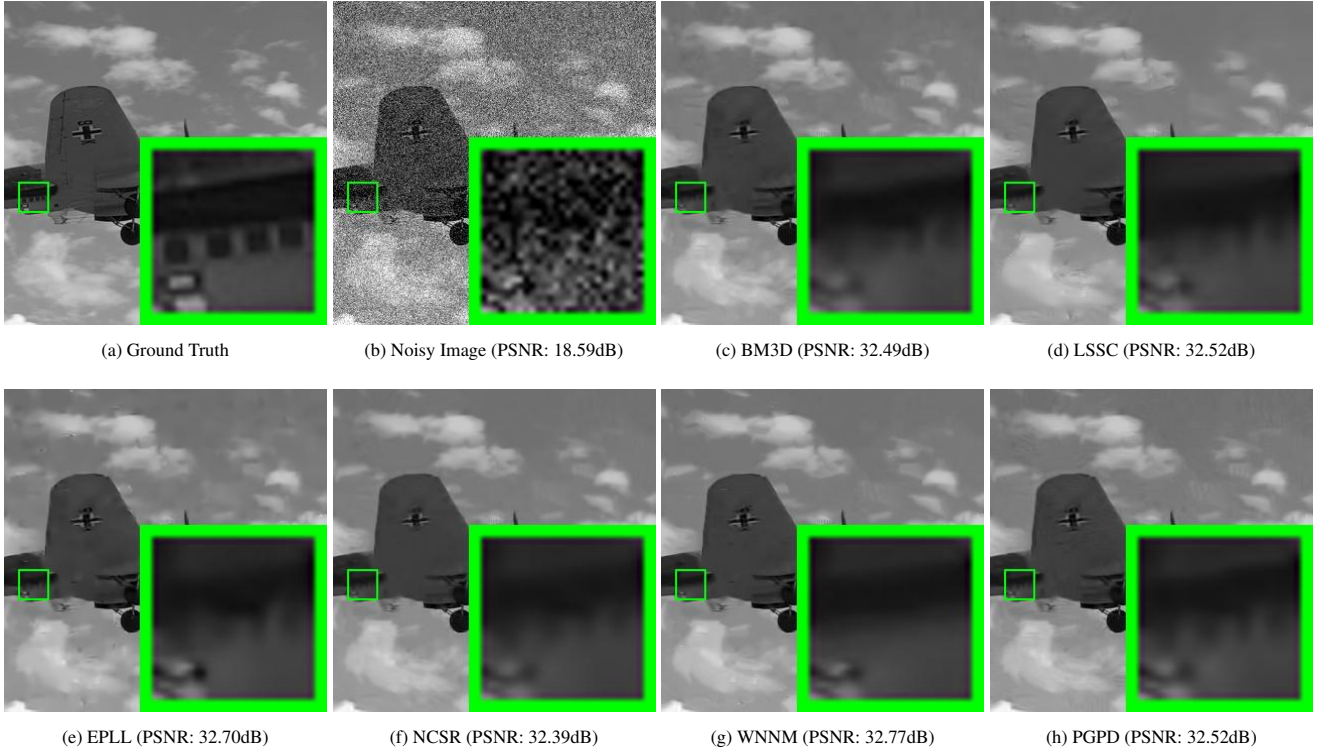


Figure 15. Denoised images of 3063 by different methods (the standard deviation of noise is $\sigma = 30$).

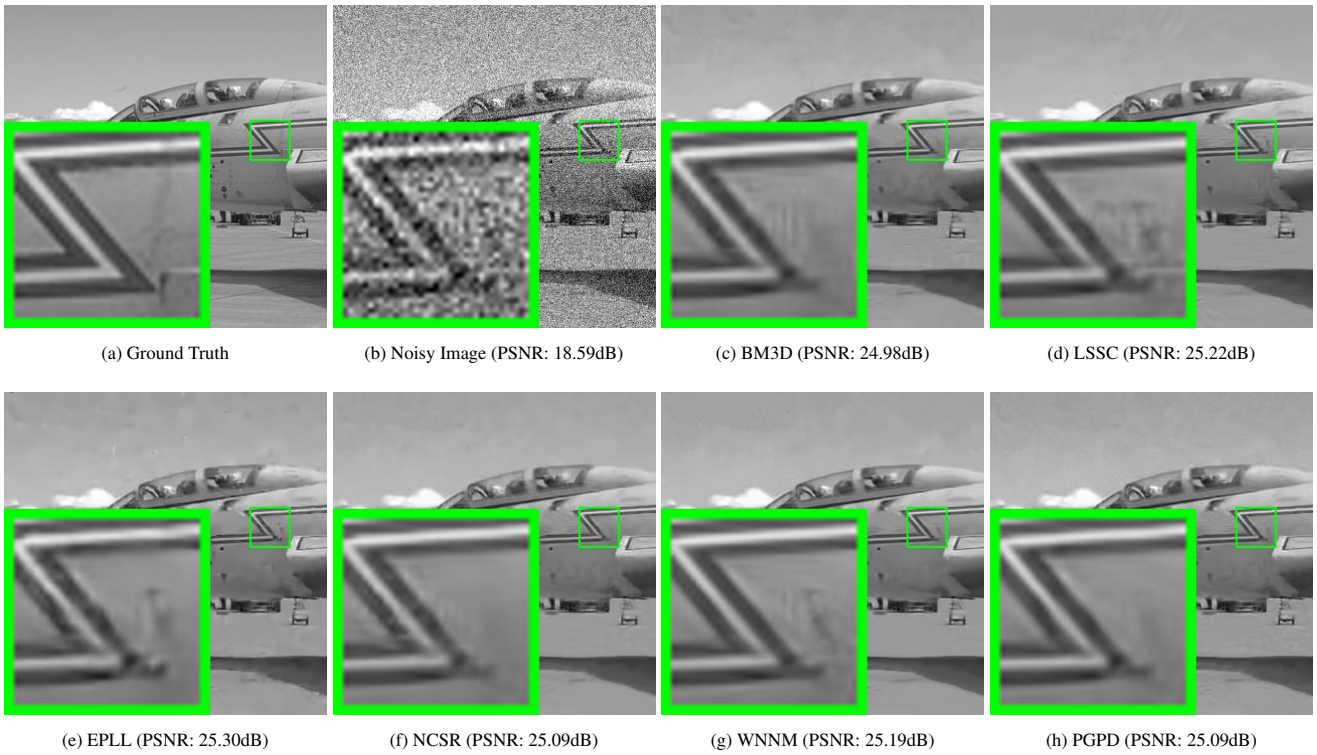


Figure 16. Denoised images of 10081 by different methods (the standard deviation of noise is $\sigma = 30$).

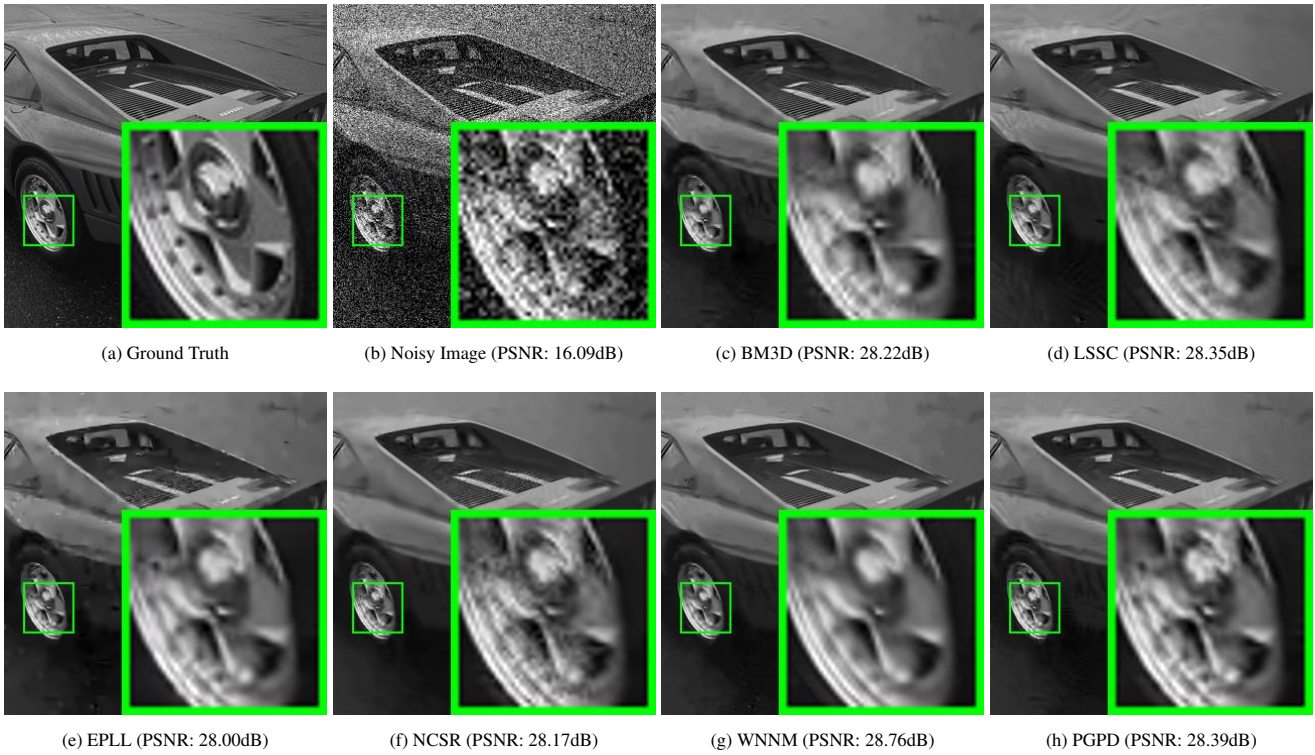


Figure 17. Denoised images of 29030 by different methods (the standard deviation of noise is $\sigma = 40$).

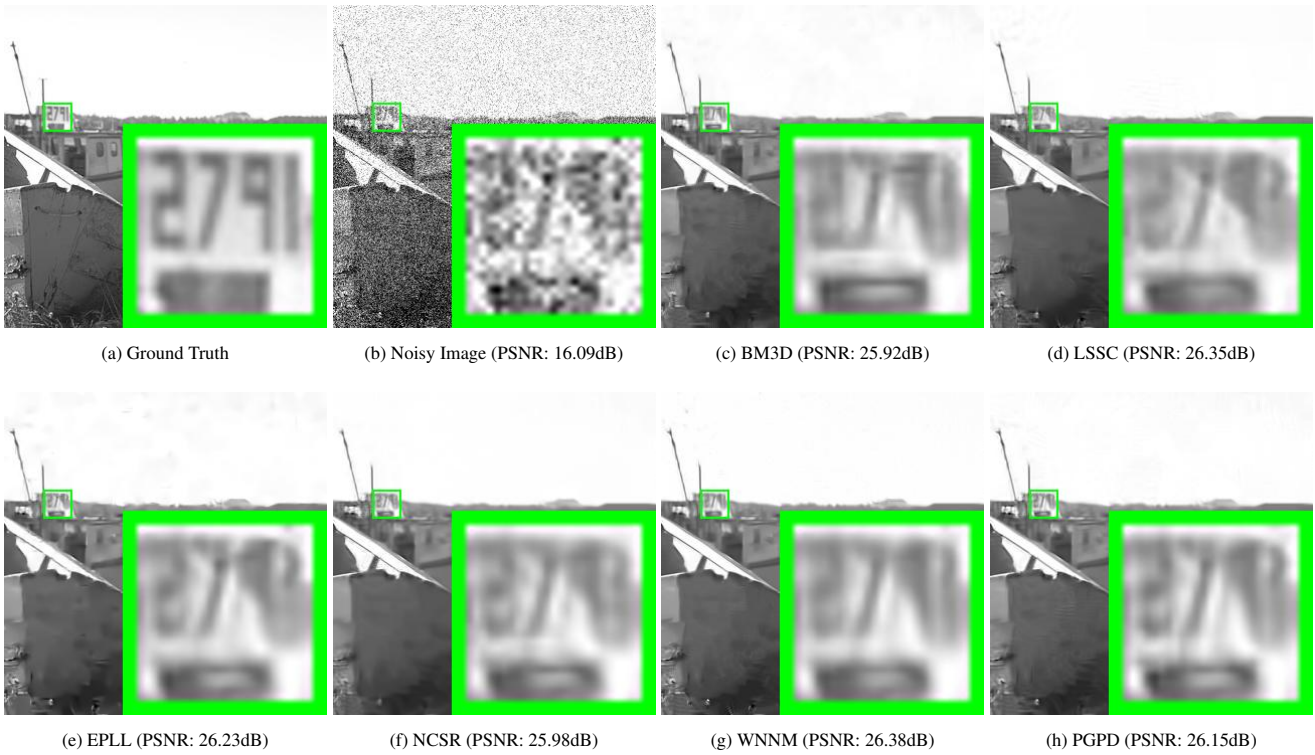


Figure 18. Denoised images of 393035 by different methods (the standard deviation of noise is $\sigma = 40$).



(a) Ground Truth (b) Noisy Image (PSNR: 14.15dB) (c) BM3D (PSNR: 24.48dB) (d) LSSC (PSNR: 24.59dB)



(e) EPLL (PSNR: 24.62dB) (f) NCSR (PSNR: 24.45dB) (g) WNNM (PSNR: 24.76dB) (h) PGPD (PSNR: 24.69dB)

Figure 19. Denoised images of 258089 by different methods (the standard deviation of noise is $\sigma = 50$).



(a) Ground Truth (b) Noisy Image (PSNR: 14.15dB) (c) BM3D (PSNR: 24.46dB) (d) LSSC (PSNR: 24.58dB)



(e) EPLL (PSNR: 24.71dB) (f) NCSR (PSNR: 24.41dB) (g) WNNM (PSNR: 24.67dB) (h) PGPD (PSNR: 24.62dB)

Figure 20. Denoised images of 253016 by different methods (the standard deviation of noise is $\sigma = 50$).

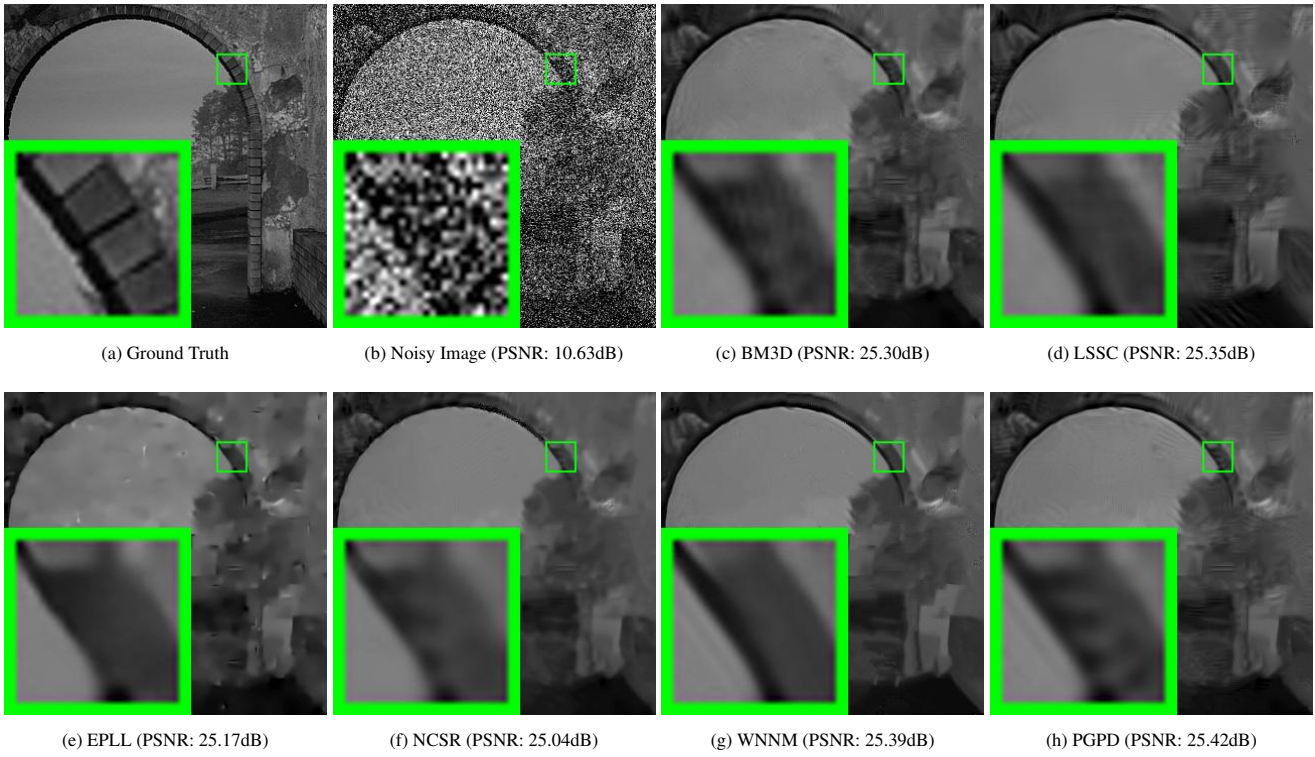


Figure 21. Denoised images of 5096 by different methods (the standard deviation of noise is $\sigma = 75$).

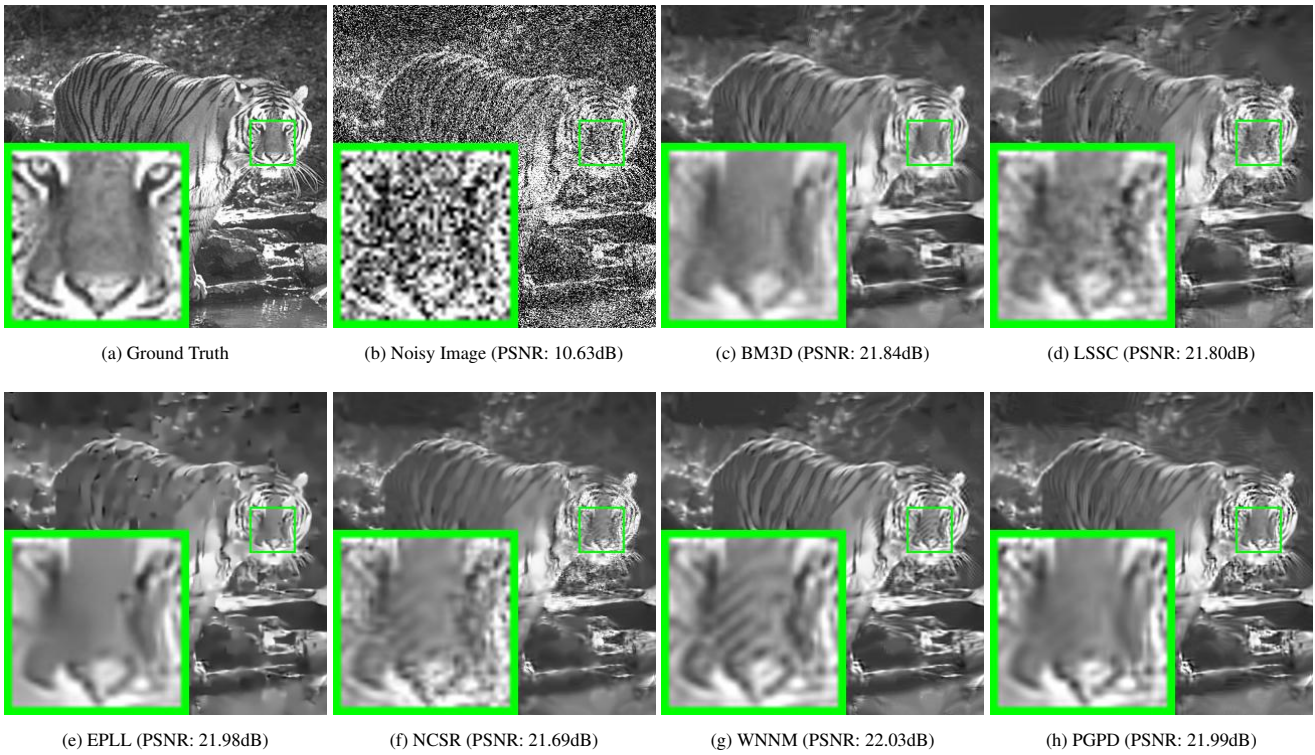


Figure 22. Denoised images of 108036 by different methods (the standard deviation of noise is $\sigma = 75$).

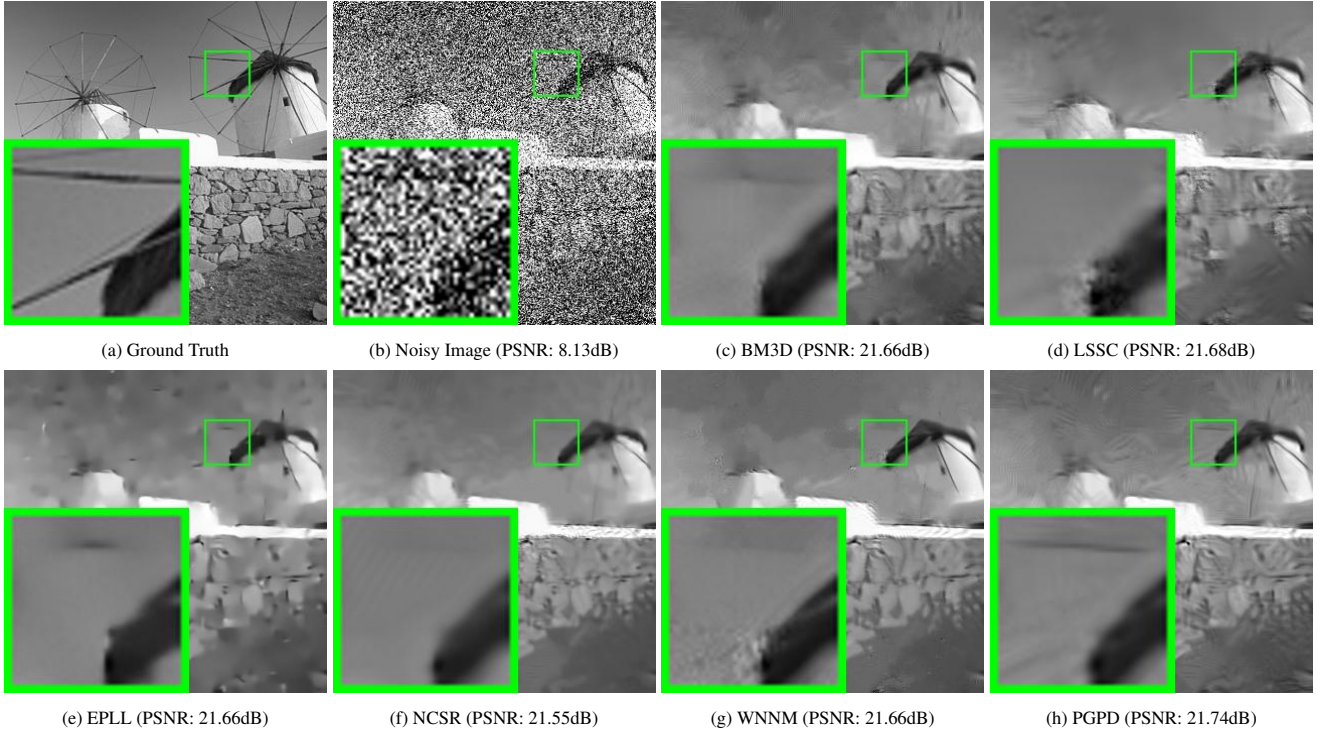


Figure 23. Denoised images of 118031 by different methods (the standard deviation of noise is $\sigma = 100$).

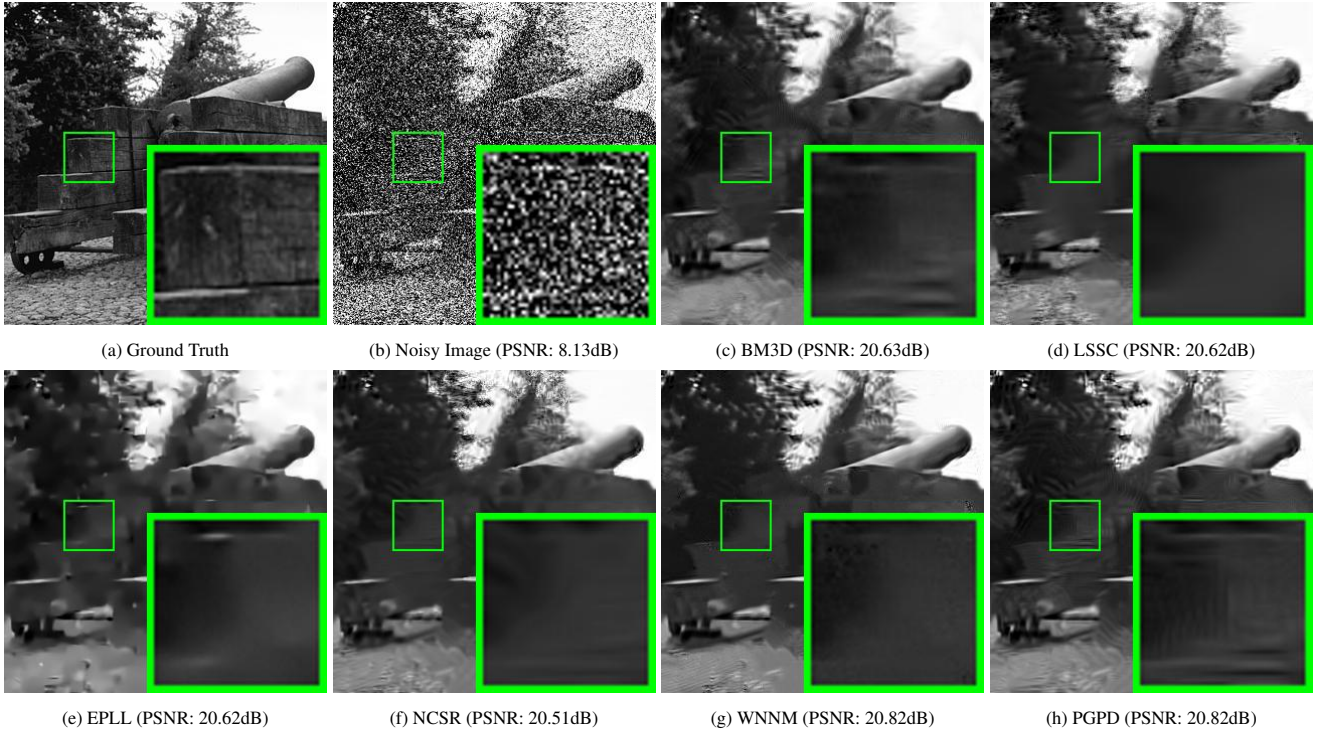


Figure 24. Denoised images of 385022 by different methods (the standard deviation of noise is $\sigma = 100$).

References

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