Multi-channel Weighted Nuclear Norm Minimization for Real Color Image Denoising

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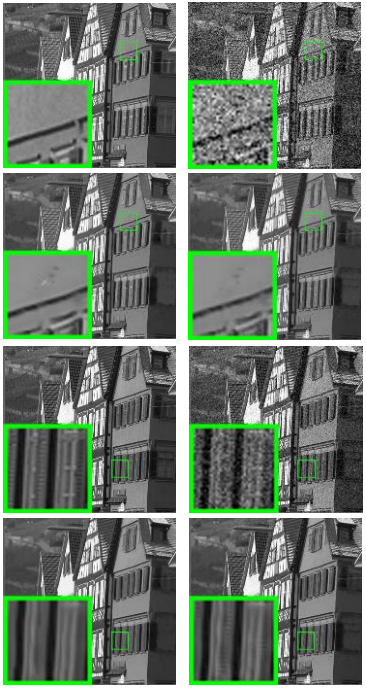
Insights

One Fact: Noise in sRGB space has different variances for different channels.

Solution: Introduce weights to balance the noise difference in different channels.



The MCWNNM Model

Model: $\min_{\mathbf{X}} \|\mathbf{W}(\mathbf{Y}-\mathbf{X})\|_F^2 + \|\mathbf{X}\|_{\boldsymbol{w},*}$

Weights: $\mathbf{W} = \begin{pmatrix} \sigma_r^{-1}\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma_g^{-1}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sigma_b^{-1}\mathbf{I} \end{pmatrix}$

Variable Splitting:

$$\min_{\mathbf{X},\mathbf{Z}} \|\mathbf{W}(\mathbf{Y}-\mathbf{X})\|_F^2 + \|\mathbf{Z}\|_{\boldsymbol{w},*} \quad \text{s.t.} \quad \mathbf{X}=\mathbf{Z}$$

Lagrangian:

$$\mathcal{L}(\mathbf{X},\mathbf{Z},\mathbf{A},\rho) = \|\mathbf{W}(\mathbf{Y}-\mathbf{X})\|_F^2 + \|\mathbf{Z}\|_{\boldsymbol{w},*} + \langle \mathbf{A}, \mathbf{X}-\mathbf{Z}\rangle + \frac{\rho}{2}\|\mathbf{X}-\mathbf{Z}\|_F^2$$

ADMM:

(1) **Update X while fixing Z and A**:

$$\mathbf{X}_{k+1} = \arg\min_{\mathbf{X}} \|\mathbf{W}(\mathbf{Y}-\mathbf{X})\|_F^2 + \frac{\rho_k}{2}\|\mathbf{X}-\mathbf{Z}_k+\rho_k^{-1}\mathbf{A}_k\|_F^2$$

(2) **Update Z while fixing X and A**:

$$\mathbf{Z}_{k+1} = \arg\min_{\mathbf{Z}} \frac{\rho_k}{2}\|\mathbf{Z}-(\mathbf{X}_{k+1}+\rho_k^{-1}\mathbf{A}_k)\|_F^2 + \|\mathbf{Z}\|_{\boldsymbol{w},*}$$

(3) **Update A while fixing X and Z**:

$$\mathbf{A}_{k+1} = \mathbf{A}_k + \rho_k(\mathbf{X}_{k+1}-\mathbf{Z}_{k+1})$$

(4) **Update** ρ_k: $\rho_{k+1} = \mu * \rho_k$, where $\mu > 1$

Convergence Guarantee:

Theorem 1. *Assume that the weights in* $\boldsymbol{w}$ *are in a non-descending order, the sequences* $\{\mathbf{X}_k\}$, $\{\mathbf{Z}_k\}$, *and* $\{\mathbf{A}_k\}$ *generated in Algorithm 1 satisfy:*

$$(a)\ \lim_{k\to\infty} \|\mathbf{X}_{k+1}-\mathbf{Z}_{k+1}\|_F = 0;$$

$$(b)\ \lim_{k\to\infty} \|\mathbf{X}_{k+1}-\mathbf{X}_k\|_F = 0;$$

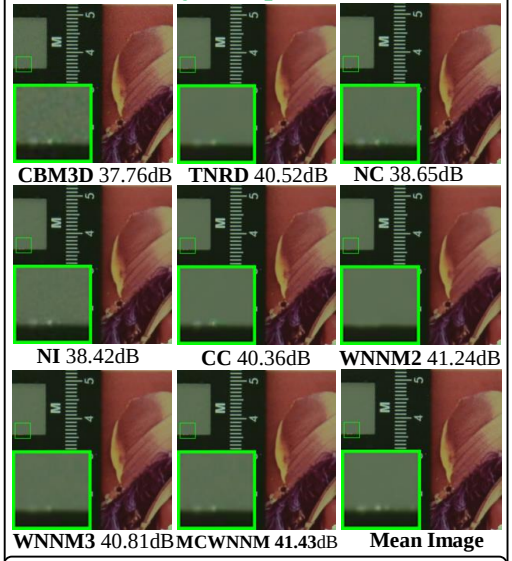
$$(c)\ \lim_{k\to\infty} \|\mathbf{Z}_{k+1}-\mathbf{Z}_k\|_F = 0.$$

Experiments

PSNR Results on Real Color Images

Camera Settings	CBM3D	MLP	TNRD	DnCNN	NI	NC	CC	WNNM-1	WNNM-2	WNNM-3	MC-WNNM
Canon 5D ISO = 3200	39.76	39.00	39.51	37.26	35.68	36.20	38.37	37.51	39.74	39.98	**41.13**
	36.40	36.34	36.47	34.13	34.03	34.35	35.37	33.86	35.12	36.65	**37.28**
	36.37	36.33	36.45	34.09	32.63	33.10	34.91	31.43	33.14	34.63	**36.52**
Nikon D600 ISO = 3200	34.18	34.70	34.79	33.62	31.78	32.28	34.98	33.46	35.08	35.08	**35.53**
	35.07	36.20	36.37	34.48	35.16	35.34	35.95	36.09	36.42	36.84	**37.02**
	37.13	39.33	39.49	35.41	39.98	40.51	**41.15**	39.86	40.78	39.24	39.56
Nikon D800 ISO = 1600	36.81	37.95	38.11	35.79	34.84	35.09	37.99	36.35	38.28	38.61	**39.26**
	37.76	40.23	40.52	36.08	38.42	38.65	40.36	39.99	41.24	40.81	**41.43**
	37.51	37.94	38.17	35.48	35.79	35.85	38.30	37.15	38.04	38.96	**39.55**
Nikon D800 ISO = 3200	35.05	37.55	37.69	34.08	38.36	38.56	39.01	38.60	**39.93**	37.97	38.91
	34.07	35.91	35.90	33.70	35.53	35.76	36.75	36.04	37.32	37.30	**37.41**
	34.42	38.15	38.21	33.31	40.05	40.59	39.06	39.73	**41.52**	38.68	39.39
Nikon D800 ISO = 6400	31.13	32.69	32.81	29.83	34.08	34.25	34.61	33.29	**35.20**	34.57	34.80
	31.22	32.33	32.33	30.55	32.13	32.38	33.21	31.16	33.61	33.43	**33.95**
	30.97	32.29	32.29	30.09	31.52	31.76	33.22	31.98	33.62	**34.02**	33.94
Average	35.19	36.46	36.61	33.86	35.33	35.65	36.88	35.77	37.27	37.12	**37.71**
Time	7.8	20.4	6.7	180.3	**0.9**	18.2	NA	689.1	465.3	198.6	202.9

Visual Quality Comparison



Matlab Code & More Details Available @ Github