Multi-channel Weighted Nuclear Norm Minimization for Real Color Image Denoising

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Insights

One Fact: Noise in sRGB space has different variances for different channels.

Solution: Introduce weights to balance the noise difference in different channels.

The MCWNNM Model

Model: \( \min_X \| W(Y - X) \|_F^2 + \| X \|_{w,*} \)

Weights:

\[
W = \begin{pmatrix}
\sigma_f^{-1}I & 0 & 0 \\
0 & \sigma_g^{-1}I & 0 \\
0 & 0 & \sigma_b^{-1}I
\end{pmatrix}
\]

Variable Splitting:

\[
\min_{X,Z} \| W(Y - X) \|_F^2 + \| Z \|_{w,*} \quad \text{s.t.} \quad X = Z
\]

Lagrangian:

\[
\mathcal{L}(X,Z,A,\rho) = \| W(Y - X) \|_F^2 + \| Z \|_{w,*} + \langle A, X - Z \rangle + \frac{\rho}{2} \| X - Z \|_F^2
\]

ADMM:

1. Update \( X \) while fixing \( Z \) and \( A \):

\[
X_{k+1} = \arg\min_X \| W(Y - X) \|_F^2 + \frac{\rho_k}{2} \| X - Z_k + \rho_k^{-1}A_k \|_F^2
\]

2. Update \( Z \) while fixing \( X \) and \( A \):

\[
Z_{k+1} = \arg\min_Z \frac{\rho_k}{2} \| Z - (X_{k+1} + \rho_k^{-1}A_k) \|_F^2 + \| Z \|_{w,*}
\]

3. Update \( A \) while fixing \( X \) and \( Z \):

\[
A_{k+1} = A_k + \rho_k(X_{k+1} - Z_{k+1})
\]

4. Update \( \rho_k \): \( \rho_{k+1} = \mu \ rho_k \), where \( \mu > 1 \)

Convergence Guarantee:

**Theorem 1.** Assume that the weights in \( w \) are in a non-descending order, the sequences \{\( X_k \), \{\( Z_k \), and \{\( A_k \)\} generated in Algorithm 1 satisfy:

(a) \( \lim_{k \to \infty} \| X_{k+1} - Z_{k+1} \|_F = 0 \);

(b) \( \lim_{k \to \infty} \| X_{k+1} - X_k \|_F = 0 \);

(c) \( \lim_{k \to \infty} \| Z_{k+1} - Z_k \|_F = 0 \).

Experiments

PSNR Results on Real Color Images

<table>
<thead>
<tr>
<th>Camera Settings</th>
<th>CBM3D</th>
<th>TNRD</th>
<th>NC</th>
</tr>
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<tbody>
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<td>40.52</td>
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<td>Nikon D3000 4</td>
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<td>38.65</td>
</tr>
</tbody>
</table>

Visual Quality Comparison

- CBM3D 37.76dB
- TNRD 40.52dB
- NC 38.65dB

Matlab Code & More Details Available @ Github