Supplementary File to “Non-local Image Smoothing with Objective Evaluation”

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In this supplementary file, we provide:
• detailed Haar transformation and inverse Haar transformation;
• more comparisons of different image smoothing methods on the datasets of NKS, [1], [6], [9].

I. DETAILED HAAR TRANSFORMATION AND INVERSE HAAR TRANSFORMATION

We perform standard Haar transformation and inverse Haar transformation with no modification. Moreover we set $q = 4$, $m = 16$ in all experiments, so the similar pixels matrix $S \in \mathbb{R}^{4 \times 16}$ could be represented by columns as $S = [s^1_1, ..., s^1_{16}] \in \mathbb{R}^{4 \times 16}$. The Haar transformation includes horizontal and vertical transformation. We first apply the horizontal transformation.

Specifically, we multiply the similar pixels matrix $R$ and the horizontal transformation matrix $H_r \in \mathbb{R}^{16 \times 16}$.

\[
\begin{align*}
t^4_i &= \frac{1}{\sqrt{16}} (\sum_{j=1}^{8} s^4_j + (-1)^{i-1} \sum_{j=9}^{16} s^4_j), \quad \text{when } i = 1, 2; \\
t^4_i &= \frac{1}{\sqrt{8}} (\sum_{j=8(1-3)+3}^{8(1-3)+4} s^4_j - \sum_{j=8(1-3)+5}^{8(1-2)} s^4_j), \quad \text{when } i = 3, 4; \\
t^4_i &= \frac{1}{\sqrt{4}} (\sum_{j=4(i-5)+2}^{4(i-5)+4} s^4_j - \sum_{j=4(i-5)+1}^{4(i-5)+3} s^4_j), \quad \text{when } i = 5, ..., 8; \\
t^4_i &= \frac{1}{\sqrt{2}} (s^4_{2(i-9)+1} - s^4_{2(i-9)+2}), \quad \text{when } i = 9, ..., 16.
\end{align*}
\]

We stack the all these column vectors to form $T = [t^4_1, ..., t^4_{16}] \in \mathbb{R}^{4 \times 16}$. We then represent $T$ by rows as $T^4 = [t^1^T, ..., t^4^T]^T \in \mathbb{R}^{16 \times 4}$, and perform vertical Haar transformation. Specifically, we multiply the matrix $T^4 \in \mathbb{R}^{4 \times 16}$ and the vertical transformation matrix $H_v \in \mathbb{R}^{4 \times 4}$.

\[
\begin{align*}
\hat{t}^1 &= \frac{1}{\sqrt{4}} \sum_{i=1}^{4} t^i, \quad \hat{t}^2 = \frac{1}{\sqrt{4}} (\sum_{i=1}^{2} t^i - \sum_{i=3}^{4} t^i), \\
\hat{t}^3 &= \frac{1}{\sqrt{2}} (t^1 - t^2), \quad \hat{t}^4 = \frac{1}{\sqrt{2}} (t^3 - t^4).
\end{align*}
\]

After the thresholding step, we could get the thresholded representation matrix $\hat{T} \in \mathbb{R}^{4 \times 16}$. We next perform inverse vertical Haar transformation and inverse horizontal Haar transformation. We first apply the inverse vertical transformation. Specifically, we multiply the inverse vertical transformation matrix $H_{vl} \in \mathbb{R}^{4 \times 4}$ and the thresholded representation matrix $T^4 \in \mathbb{R}^{4 \times 16}$.

\[
\begin{align*}
\hat{t}^1 &= \frac{1}{\sqrt{4}} (\hat{t}^1 + \hat{t}^2) + \frac{1}{\sqrt{2}} \hat{t}^3, \\
\hat{t}^2 &= \frac{1}{\sqrt{4}} (\hat{t}^1 + \hat{t}^2) - \frac{1}{\sqrt{2}} \hat{t}^3, \\
\hat{t}^3 &= \frac{1}{\sqrt{4}} (\hat{t}^1 - \hat{t}^2), \\
\hat{t}^4 &= \frac{1}{\sqrt{4}} (\hat{t}^1 - \hat{t}^2) - \frac{1}{\sqrt{2}} \hat{t}^3.
\end{align*}
\]

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We stack all these row vectors to form $\tilde{T}^4 = [(\tilde{t}^1)^T, ..., (\tilde{t}^4)^T]^T \in \mathbb{R}^{4 \times 16}$. We then represent $\tilde{T}$ by columns as $\tilde{T} = [\tilde{t}_1, ..., \tilde{t}_{16}] \in \mathbb{R}^{4 \times 16}$, and perform inverse horizontal Haar transformation. Specifically, we multiply the matrix $\tilde{T} \in \mathbb{R}^{4 \times 16}$ and the inverse horizontal transformation matrix $H_{ir} \in \mathbb{R}^{16 \times 16}$:

$$
\begin{align*}
\tilde{s}_1^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 + \tilde{t}_2^4) + \frac{1}{\sqrt{8}} \tilde{t}_3^4 + \frac{1}{4} \tilde{t}_5^4 + \frac{1}{\sqrt{2}} \tilde{t}_9^4, \\
\tilde{s}_2^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 + \tilde{t}_2^4) + \frac{1}{\sqrt{8}} \tilde{t}_3^4 + \frac{1}{4} \tilde{t}_5^4 - \frac{1}{\sqrt{2}} \tilde{t}_9^4, \\
\tilde{s}_3^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 + \tilde{t}_2^4) + \frac{1}{\sqrt{8}} \tilde{t}_3^4 - \frac{1}{4} \tilde{t}_5^4 + \frac{1}{\sqrt{2}} \tilde{t}_{10}^4, \\
\tilde{s}_4^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 + \tilde{t}_2^4) + \frac{1}{\sqrt{8}} \tilde{t}_3^4 - \frac{1}{4} \tilde{t}_5^4 - \frac{1}{\sqrt{2}} \tilde{t}_{10}^4, \\
\tilde{s}_5^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 + \tilde{t}_2^4) - \frac{1}{\sqrt{8}} \tilde{t}_3^4 + \frac{1}{4} \tilde{t}_5^4 + \frac{1}{\sqrt{2}} \tilde{t}_{11}^4, \\
\tilde{s}_6^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 + \tilde{t}_2^4) - \frac{1}{\sqrt{8}} \tilde{t}_3^4 + \frac{1}{4} \tilde{t}_5^4 - \frac{1}{\sqrt{2}} \tilde{t}_{11}^4, \\
\tilde{s}_7^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 + \tilde{t}_2^4) - \frac{1}{\sqrt{8}} \tilde{t}_3^4 - \frac{1}{4} \tilde{t}_5^4 + \frac{1}{\sqrt{2}} \tilde{t}_{12}^4, \\
\tilde{s}_8^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 + \tilde{t}_2^4) - \frac{1}{\sqrt{8}} \tilde{t}_3^4 - \frac{1}{4} \tilde{t}_5^4 - \frac{1}{\sqrt{2}} \tilde{t}_{12}^4, \\
\tilde{s}_9^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 - \tilde{t}_2^4) + \frac{1}{\sqrt{8}} \tilde{t}_4^4 + \frac{1}{4} \tilde{t}_6^4 + \frac{1}{\sqrt{2}} \tilde{t}_{13}^4, \\
\tilde{s}_{10}^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 - \tilde{t}_2^4) + \frac{1}{\sqrt{8}} \tilde{t}_4^4 + \frac{1}{4} \tilde{t}_6^4 - \frac{1}{\sqrt{2}} \tilde{t}_{13}^4, \\
\tilde{s}_{11}^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 - \tilde{t}_2^4) + \frac{1}{\sqrt{8}} \tilde{t}_4^4 - \frac{1}{4} \tilde{t}_6^4 + \frac{1}{\sqrt{2}} \tilde{t}_{14}^4, \\
\tilde{s}_{12}^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 - \tilde{t}_2^4) + \frac{1}{\sqrt{8}} \tilde{t}_4^4 - \frac{1}{4} \tilde{t}_6^4 - \frac{1}{\sqrt{2}} \tilde{t}_{14}^4, \\
\tilde{s}_{13}^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 - \tilde{t}_2^4) - \frac{1}{\sqrt{8}} \tilde{t}_4^4 + \frac{1}{4} \tilde{t}_6^4 + \frac{1}{\sqrt{2}} \tilde{t}_{15}^4, \\
\tilde{s}_{14}^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 - \tilde{t}_2^4) - \frac{1}{\sqrt{8}} \tilde{t}_4^4 + \frac{1}{4} \tilde{t}_6^4 - \frac{1}{\sqrt{2}} \tilde{t}_{15}^4, \\
\tilde{s}_{15}^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 - \tilde{t}_2^4) - \frac{1}{\sqrt{8}} \tilde{t}_4^4 - \frac{1}{4} \tilde{t}_6^4 + \frac{1}{\sqrt{2}} \tilde{t}_{16}^4, \\
\tilde{s}_{16}^4 &= \frac{1}{\sqrt{16}}(\tilde{t}_1^4 - \tilde{t}_2^4) - \frac{1}{\sqrt{8}} \tilde{t}_4^4 - \frac{1}{4} \tilde{t}_6^4 - \frac{1}{\sqrt{2}} \tilde{t}_{16}^4.
\end{align*}
$$

We stack all these column vectors together and form the smoothed similar pixel matrix $\tilde{S} = [\tilde{s}_1^4, ..., \tilde{s}_{16}^4] \in \mathbb{R}^{4 \times 16}$.

## II. More Comparisons of Different Image Smoothing Methods

Here, we conduct more comparisons of different image smoothing methods on the datasets of NKS, [1], [6], [9]. In Figures 1-5, we compare PSNR, SSIM [4], FSIM [7], and visual quality of different methods on image smoothing on the dataset of NKS. In Figures 6-24, we compare the visual quality of different methods on image smoothing on the datasets of [1], [6], [9]. The comparison results demonstrate that the PNLS method achieves better visual quality than the other image smoothing methods.
Fig. 1. Comparison of smoothed images and PSNR(dB)/SSIM/FSIM results by different methods on the image “S03_T07” from our NKS dataset. The best results are highlighted in bold.

Fig. 2. Comparison of smoothed images and PSNR(dB)/SSIM/FSIM results by different methods on the image “S07_T02” from our NKS dataset. The best results are highlighted in bold.

Fig. 3. Comparison of smoothed images and PSNR(dB)/SSIM/FSIM results by different methods on the image “S09_T09” from our NKS dataset. The best results are highlighted in bold.
Fig. 4. Comparison of smoothed images and PSNR(dB)/SSIM/FSIM results by different methods on the image “S10_T02” from our NKS dataset. The best results are highlighted in **bold**.

Fig. 5. Comparison of smoothed images and PSNR(dB)/SSIM/FSIM results by different methods on the image “S14_T06” from our NKS dataset. The best results are highlighted in **bold**.

Fig. 6. Comparison of smoothed images by different methods on the image “0073” from the DIV2K dataset [1].
Fig. 7. Comparison of smoothed images by different methods on the image "0102" from the DIV2K dataset [1].

Fig. 8. Comparison of smoothed images by different methods on the image "0105" from the DIV2K dataset [1].

Fig. 9. Comparison of smoothed images by different methods on the image "0117" from the DIV2K dataset [1].
Fig. 10. Comparison of smoothed images by different methods on the image “0146” from the DIV2K dataset [1].

Fig. 11. Comparison of smoothed images by different methods on the image “0154” from the DIV2K dataset [1].

Fig. 12. Comparison of smoothed images by different methods on the image “0166” from the DIV2K dataset [1].
Fig. 13. Comparison of smoothed images by different methods on the image “0205” from the DIV2K dataset [1].

Fig. 14. Comparison of smoothed images by different methods on the image “0404” from the DIV2K dataset [1].

Fig. 15. Comparison of smoothed images by different methods on the image “0094” from the dataset in [9].
Fig. 16. Comparison of smoothed images by different methods on the image “0115” from the dataset in [9].

Fig. 17. Comparison of smoothed images by different methods on the image “0169” from the dataset in [9].
Fig. 18. Comparison of smoothed images by different methods on the image “0314” from the dataset in [9].

Fig. 19. Comparison of smoothed images by different methods on the image “0334” from the dataset in [9].
Fig. 20. Comparison of smoothed images by different methods on the image “11_07” from the dataset in [6]

Fig. 21. Comparison of smoothed images by different methods on the image “11_08” from the dataset in [6]
Fig. 22. Comparison of smoothed images by different methods on the image “11_11” from the dataset in [6].

Fig. 23. Comparison of smoothed images by different methods on the image “11_26” from the dataset in [6].
Fig. 24. Comparison of smoothed images by different methods on the image “12_53” from the dataset in [6]
REFERENCES


